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FUNDAMENTAL LIMITATIONS TO OPTICAL DOPPLER MEASUREMENTS FOR SPACE NAVIGATION

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FIGURES

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SUMMARY

Theoretical consideration is given to the problem of optical Doppler velocity determination of the high accuracy (about 1 ft/sec) required to be useful for space navigation. From the physical theory of line-broadening and -shifting mechanisms in stellar atmospheres, it is concluded that an intrinsic variability of ±200 ft/sec may be expected in the measurement of an observer's Doppler velocity. Examination of the current state of the art in measuring equipment suggests that it does not set the limits on accuracy.

I. INTRODUCTION

It has been suggested that a useful portion of the information needed to make orbit corrections for a spacecraft undergoing the maneuvers of midcourse or approach guidance can be obtained from the determination of the spacecraft's velocity,

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by measuring the Doppler shift of absorption lines in the spectra of astronomical objects.[1] Noton has shown that the present state of the art for injection guidance is sufficient to permit the treatment of departures of the spacecraft's true orbit from the standard trajectory by linear perturbation theory.[2] He computed that an error in the range rate determination of 1 ft/sec during midcourse guidance would lead to a 4000-mile miss at Mars. Hence, velocity information, in order to be useful, must be accurate to about 1 ft/sec. For a representative wavelength of 4000 angstroms, the corresponding Doppler shift is 4×10^{-6} angstroms.

The physical mechanisms affecting the shapes and positions of spectral lines will be discussed, and limitations on the accuracy of observed Doppler velocities will be found to be caused by the inconstancy of these mechanisms. Of the available celestial electromagnetic sources to be considered, the only practical one is the Sun, since the flux from the brightest star is only 10^{-10} that from the Sun.

In addition, present spectroscopic techniques and instrumentation will be discussed, with emphasis upon the accuracy obtainable within the state of the art as imposed by the restrictions encountered in space-borne missions.

II. PHYSICAL NATURE OF STELLAR ABSORPTION LINES IN THE OPTICAL REGION

Absorption lines in stellar spectra, as for example in the spectrum of the Sun, are fundamentally affected by the physical conditions existing in the region of the stellar atmosphere where the absorption takes place.

A. Absorption-Line Broadening

The general result of these effects is to broaden an otherwise monochromatic absorption line into a feature from which the physical conditions in the atmosphere may be induced. Let us discuss briefly the physical nature of these broadening mechanisms and the effect which they have upon the absorption line. The intrinsic broadening mechanisms are:

1. Doppler effect, arising from the random kinetic motions of atoms. This is a direct consequence of their kinetic temperature, and to it must be added "turbulent" broadening because of large scale motions of large masses of gas in the star's atmosphere. While Doppler broadening produces a symmetric absorption line, turbulent broadening sometimes does not, and is especially dominant in the atmospheres of giant and supergiant stars.
2. Radiation damping, which is a consequence of the finite lifetimes of excited energy levels of atoms by way of the Heisenberg uncertainty principle. Classically, it corresponds to the fact that the finite wave train emitted by a radiating atom is nonmonochromatic.
3. Collision damping. In radiation damping, the lifetime parameter referred to is an atomic constant, equal to the reciprocal of the sum of the Einstein probability-of-spontaneous-emission-per-unit-time coefficients over all possible lower energy levels into which the atom may de-excite. A radiating atom may be perturbed by its neighbors so that its lifetime is reduced.

4. Stark effect, caused by the statistically fluctuating electric fields produced by ions and electrons in the neighborhood of the radiating atom.
5. Hyperfine structure, arising from the possible angular momentum states of the nucleus, is responsible for the broadening of certain lines in the solar spectrum.
6. Zeeman effect. Lines produced in sunspots or magnetic stars are broadened or split into components by the magnetic field.

In addition, there are extrinsic causes of line broadening in stellar spectra. A spectral line from a rapidly rotating star is made up of contributions from all points on the disk, parts of which are approaching and parts of which are receding. The net result is an absorption line with a broad, dish-shaped profile. Expanding shells around novae, peculiar stars, and variable stars such as Cepheids also produce abnormally broadened lines. Any phenomenon which destroys the equality of the two limbs of a rotating star in contributing to the light received by an observer will result in a Doppler shift of the line center. An obvious violator in such a circumstance is a sunspot group.

For our purposes, we are not concerned directly with the shape, width, or depth of a particular absorption line; we wish to determine the shift in the line due to the relative velocity between the observer and the source. To determine this to an accuracy of 1 ft/sec requires that we investigate the sources of line broadening listed above, for if any of the broadening is asymmetric, a shift in the centroid of the line will be introduced. The centroid is defined as that wavelength seen by the observer such that one-half of the radiant energy absorbed from the continuum lies to the red and the other half to the blue. Thus, for an asymmetric line the centroid will not coincide exactly with the minimum or point of greatest absorption in the line. To put it another way, consider a source at rest with respect to the observer. If any of the above mechanisms broaden the line on one side of the central (zero velocity) wave length more than the other, to the observer, who is recording the centroid of the line, it will appear that the line is slightly shifted, indicating some relative velocity between him and the source. These physical limitations will introduce a bias in the measurements, and it will be well to investigate whether this bias is of a constant nature in time. If it is not, then a natural variability in the measured Doppler velocity is to be expected which cannot be removed.

Having established that it is only those broadening mechanisms which produce asymmetry that we wish to consider, we can immediately dismiss those that are symmetric. They are (numbering as before):

1. Doppler broadening, characterized by a bell-shaped absorption coefficient $\exp\left[- \text{const } (\Delta\lambda)^2\right]$. Figure 1a illustrates the line profile of Ca II, λ_0 = 3934 angstroms, for varying abundances in the solar atmosphere.

2, 3. Radiation and collisional broadening, characterized by an absorption coefficient with a deep, narrow core and broad wings, in accordance with the dispersion formula $A/\left[B + (\Delta\lambda)^2\right]$. Figure 1b shows the line profile for radiation damping only, the material being 1.8×10^{18} atom/gm of Ca II, λ_0 = 3934 angstroms. Collision broadening gives a profile similar to radiation damping.

Figure 2 shows the line profile (computed and observed) for the Ca II K-line in the Sun, with $T = 5700^0$ K and $N = 1.8 \times 10^{18}$ atom/gm.

4. Linear Stark effect, which produces components spaced symmetrically about the unperturbed line center.
5. Hyperfine structure, which, though not necessarily symmetric, is the same for a reference laboratory source as for a stellar source.
6. Linear Zeeman effect, which can be derived from a linear perturbation of the Hamiltonian in Schroedinger's equation, is symmetric for weak magnetic fields, both in line splitting and in intensity distribution.
7. (Extrinsic). Rotational broadening, from rapidly rotating stars, which produces symmetric broadening.

This leaves the following asymmetry-producing mechanisms to consider:

1. Quadratic Stark effect, which arises because the presence of an external electric field in addition to the coulomb field of the nucleus changes the energy levels of the electrons, and the change is not the same for all levels. Another more complicated form of the quadratic Stark effect arises from the presence of neutral atoms of other species and is known as "pressure shift".
2. Second-order Zeeman effect, which, due to a diamagnetic term in the perturbed Hamiltonian, produces a shift in the centroid relative to that of the unperturbed line.

3. Variable stars, as for example pulsating Cepheids, which have periodic or semi-periodic changes in radius, resulting in a true Doppler shift.
4. "Sunspot effect", already described.

B. Asymmetric Broadening Mechanisms

There is no direct information on the variation of average solar electric fields, but if we adopt, as a useful average field, the field of a single ion at a distance equal to the mean separation of particles at the mean level of absorption in the solar atmosphere and allow a relative variation in this field equal to the relative variation observed in the general solar magnetic field, we obtain a variation of approximately 1 to 9 volts/cm. The interaction energy in a hydrogen-like atom is given by[3]

$$\Delta T = 6.42 \times 10^{-5} F + 5.22 \times 10^{-16} F^2 + 1.53 \times 10^{-25} F^3 + \ldots \quad (1)$$

where ΔT represents the shift in the energy level from the field-free state due to the presence of the electric field, and F is the field strength in volts/cm.

The first observations of the second-order Stark effect in hydrogen were made by Takamine and Kokubu in the Stark pattern of $H\gamma$ when the spectrum was produced in a field of 147,000 volts/cm., and they observed a shift to the red of the middle component of $H\gamma$ of 0.8 angstroms.[4] Therefore we would expect for the assumed variation of the general solar electric field a maximum shift of the centroid wavelength of H of 3×10^{-9} angstroms. It appears that the quadratic Stark effect, at least in this form, will not prevent measurements of optical Doppler velocities to 1 ft/sec.

However, the other form of quadratic Stark effect, known as pressure shift, has a much more appreciable effect. Due to the polarization of one atom by the close approach of another, an excited or outer state will be lowered more than a tightly bound lower state. The frequency distribution during this time of close approach is added to the collision damping distribution, with the result that the observed spectral line is spread out more on the long wavelength side than on the short. The shift observed depends on the foreign gas used and is also proportional to the relative density, defined as the ratio of the density under existing conditions to the density at STP.

For an order of magnitude example, consider as a typical line the mercury resonance line $\lambda = 2537$ angstroms. The empirical pressure shift relationship with H_2 as the foreign gas (atomic hydrogen would be the most abundant foreign gas in all but the hottest stars) is:[5]

$$\Delta\lambda = 0.004\left[\frac{\rho}{\rho_0}\right] \quad (2)$$

where $\Delta\lambda$ is in angstroms and ρ_0 = density at STP.

If we assume that this relationship holds for atomic hydrogen as the foreign gas and extrapolate to densities in the solar atmosphere, taking as an example the density at about the level of formation of a moderately strong line from the model solar atmosphere of Chandrasekhar and Munch[6]:

for $\tau = 0.3$;

$$P_g = \text{antilog } 4.67 = 4.68 \times 10^4 \text{ dyne/cm}^2$$

$$T = 5850^\circ \text{ K}$$

$$\frac{\rho}{\rho_0} = \frac{P}{P_0}\frac{T}{T_0} = 2.2 \times 10^{-3}$$

$$\Delta\lambda = 9 \times 10^{-6} \text{ angstroms}$$

We must also consider the variability of this effect. Flares, sunspots, plages, and other phenomena exhibiting markedly different densities from their surroundings will have profound effects over limited regions of the Sun, but they are most unpredictable. In addition, for that small part of the absorption line formed in the chromosphere or outer atmosphere, the variation of the spicule pattern, those hot and dense spire-like cells of photospheric material thrusting up into the chromosphere, will have an appreciable effect. Also, the granule pattern in the photosphere, which is closer to the mean level of absorption, is not constant. It is therefore estimated that the above shift can vary by about its own magnitude, or produce a variability in the observed velocity of 0 to 4 ft/sec.

Let us now consider the second-order Zeeman effect. A linear perturbation of the Hamiltonian in Schroedinger's equation yields, for light circularly polarized perpendicular to the magnetic field:

$$\nu = \nu_0 \pm \frac{eH}{4\pi mc^2} + \frac{e^2H^2a_0n^4}{4mc^3} \tag{3}$$

The second-order term, giving the shift in the line centroid relative to the unperturbed line centroid, is:

$$\Delta\nu = \frac{e^2 H^2 a_0 n^4}{4mc^3} = 1.24 \times 10^{-32} H^2 \tag{4}$$

where $\Delta\nu$ is in wavenumbers, cm^{-1}, and all physical constants are in electrostatic cgs units. H is the magnetic field strength in oersteds, and n is the refractive index in the solar atmosphere. It is obvious that no reasonable variation in the general solar magnetic field will produce a centroid shift sufficient to preclude optical Doppler velocity measurement accurate to 1 ft/sec.

Turning now to the variability of stars, the third mechanism, we can, upon making simple assumptions, estimate the contribution of this effect to the bias error in a measured optical Doppler velocity. At present, it is not possible to measure the brightness of a star to better than about 1%, and it has been suggested that perhaps all stars are variable to some degree, the vast majority having a variability of less than 1% in brightness. When a star varies in brightness, its radius and temperature change, giving rise to a true Doppler shift in its spectrum. The change in radius and temperature also have profound effects upon the pressure and density in its outer layers, in turn affecting all line broadening and shifting mechanisms which operate at the interatomic level.

Let us estimate the magnitude of the Doppler shift due to the change of radius of a star similar in mass, radius, and temperature to the Sun. For such a small variation, less than 1%, we may assume that the pulsation is an adiabatic process; further, we will assume the perfect gas law, and these assumptions are embodied in the two relations:

$$P = K\rho^{\gamma}$$

$$P = \frac{k}{\mu H}\,\rho T$$

where K = constant, γ = ratio of specific heats (5/3 for a monatomic perfect gas), μ = mean molecular weight of material in units of the mass of a hydrogen atom, and H = mass of hydrogen atom. We will also employ the relation between the luminosity, radius, and effective temperature of a star:

$$L = 4\pi R^2\,\sigma T^4 \tag{6}$$

For a variation in brightness $\frac{\delta L}{L}$, we have:

$$\frac{\delta L}{L} = 2\frac{\delta R}{R} + 4\frac{\delta T}{T} \tag{7}$$

It can easily be shown that the temperature and radius variations for a monatomic perfect gas undergoing an adiabatic pulsation is given by:

$$\frac{\delta T}{T} = -\,2\frac{\delta R}{R} \tag{8}$$

Hence

$$\frac{\delta L}{L} = -\,6\frac{\delta R}{R} \tag{9}$$

If, for a star like the Sun, the brightness changes by 0.1% the radius variation will be

$$R = 1.2 \times 10^{11} \text{cm} = 3.8 \times 10^{9} \text{feet.}$$

The theory of adiabatic pulsations predicts that the product of the period of pulsation and the square root of the mean density of the star be a constant, and this is substantiated by observation. If we adopt the value of this constant as determined observationally, we have:[7]

$$p\left[\frac{\bar{\rho}}{\bar{\rho}_{\odot}}\right]^{1/2} = 0.04 \tag{10}$$

where p = period in days, and $\bar{\rho}_{\odot}$ = mean density of Sun. Thus, for a star similar to the Sun, the period of pulsation is approximately one hour. It then follows that the mean velocity of the pulsation wave will be 210 feet/sec. The observed shift in the spectral lines, however, is made up of contributions from all points on the disk, so that the observed velocity will be related to the pulsation wave velocity by:

$$v_{obs} = 0.707\ v \tag{11}$$

Here we have also included the effects of limb darkening.

Thus we may expect, if the Sun has a variation in brightness of only 0.1%, a mean shift in an observer's measured velocity of ±150 ft/sec. Of course, the maximum velocity of the pulsation will be higher than the mean velocity.

Table 1

Effect	Expected Maximum Optical Doppler Variation
Second Order Zeeman Effect	10^{-24} ft/sec
Quadratic Stark Effect	10^{-3} "
Pressure Shift	4 "
Sunspot Effect	37 "
Variability of Radius	150* "

It must be emphasized here that the values given in the tabulation represent the maximum expected Doppler variations, under certain assumptions. In the case of the variability in the luminosity of the Sun, it may be argued that the Sun is not variable. However, we are here dealing with a change in brightness an order of magnitude less than the precision with which the observations can at present be made. This phenomenon has such a profound influence on the accuracy of optical Doppler velocity measurements that the question of its existence should provide sufficient heuristic grounds for a program whose objectives are to settle the issue.

Minimum values of the expected Doppler variations for each phenomenon can be set equal to zero, but the conditions necessary to obtain minimum variations are considered extremely improbable.

*Mean value given; maximum value incalculable, since explicit mathematical form of variation is not presently known.

The fourth source of line asymmetry to be considered is the "sunspot effect". The largest sunspot group ever observed covered 0.0054 of the visible solar surface.[8] The maximum effect of such a spot group as the one above can be estimated from the following simple model. The best compromise, in the maximization of line shift, between the component of velocity in the observer's line of sight and the projected area of the spot group in the observer's direction, places the spot group 0.707 of the way from the center of the disk to the limb. The radial velocity here, at the equator, is 6900 ft/sec. We assume the solar disk to be evenly illuminated; in fact it shows pronounced limb darkening, but not enough to effect order of magnitude estimates. By the same token we assume the sunspots to be completely black; we can compute the effect of the sunspot group on the centroid wavelength due to Doppler effect alone. It will be the zero shift of the vast majority of the disk weighted by its intensity, which would be $\left[1 - 2(0.0054)\right]$ plus the shift of 6900 ft/sec of the portion of the disk on the opposite side of the disk from the sunspot group, which the group fails to balance in oppositely shifted radiation, weighted by its intensity of 0.0054. The shift of the centroid of the line in the emergent solar flux is ±37 ft/sec.

The effects of the foregoing mechanism are summarized in Table 1, from which one may conclude that a natural variation in the measurement of a spacecraft's optical Doppler velocity of approximately ±200 ft/sec may be expected.

III. INSTRUMENTATION STATE OF THE ART

The state of the art in optical Doppler shift measurements today is exemplified by the Babcock magnetograph at the Mt. Wilson observatory. A 17-inch image of the Sun is formed by an off-axis Cassegrain reflecting system with a focal length of 150 feet located in a large steel tower.[9] The spectrograph, a vertical Littrow instrument employing a large plane grating, is located in a 75-foot pit under the tower, obtaining a dispersion in the green of 11 mm/angstrom with a 75-foot-focal-length lens. The grating, used in the fifth order, has a ruled area of 5 1/4 x 8 inches, with 15,000 grooves per inch and a resolving power in the green of 600,000. Using the Rayleigh criterion for resolution, two sharp lines 0.009 angstroms apart may just be resolved in the green, but by working into the diffraction pattern much better precision is obtained in centroid measurements.

In front of the entrance slit to the spectrograph is placed an electro-optic retardation plate in the form of a Z-cut crystal of ammonium dihydrogen phosphate. Upon the application of an alternating voltage the retardation oscillates at the applied frequency, 120 cps, between plus and minus a quarter wave. A Nicol prism follows this plate, the two fixed elements thus constituting an oscillating circular analyzer.

Two slits are used at the exit of the spectrograph just before the detectors, each slit being placed at the point of steepest slope on each side of a spectral line. Thus, as the retardation of the quarter wave plate is oscillated, one measures the difference between the two exit slits first for the right hand and then the left hand circularly polarized components of solar line radiation. Magnetic fields of one gauss, causing a Zeeman splitting of 10^{-5} angstroms, can be measured accurately, and the rms shot noise of the system corresponds to about 0.1 gauss.

However, it is readily apparent that such an instrument as the Babcock magnetograph, capable of measuring an optical Doppler velocity with a precision of perhaps 3 to 5 ft/sec, is hardly practical from the standpoint of space-borne missions, because of its enormous weight and size. Limitations of instruments have been discussed by Franklin and Birx[1]; the best velocity resolution to be hoped for on the basis of detector noise considerations was 60 ft/sec. Possibilities exist for the use of interferometric techniques in order to obtain high dispersion and narrow instrumental profiles with non-bulky equipment; such an instrument might consist of a fixed-plate Fabry-Perot interferometer crossed with a small spectrograph, whose function is to separate the orders. It is interesting to note, however, that the original Frabry-Perot interferometer designed for the Babcock magnetograph was replaced by the present plane grating spectrograph because it was found impossible to maintain the temperature equilibrium necessary. Such an instrument would have to withstand the rigors of space-borne guidance maneuvers.

IV. CONCLUSIONS

Calculations have been made on the assumption that the physical characteristics of stellar electromagnetic sources, as for example the Sun, vary in such a manner that a variability of approximately ±200 ft/sec may be expected in the observer's measured optical Doppler velocity. It is the authors' opinion that it is this that imposes a fundamental limitation on the accuracy with which Doppler measurements may be made optically, rather than the instrumentation.

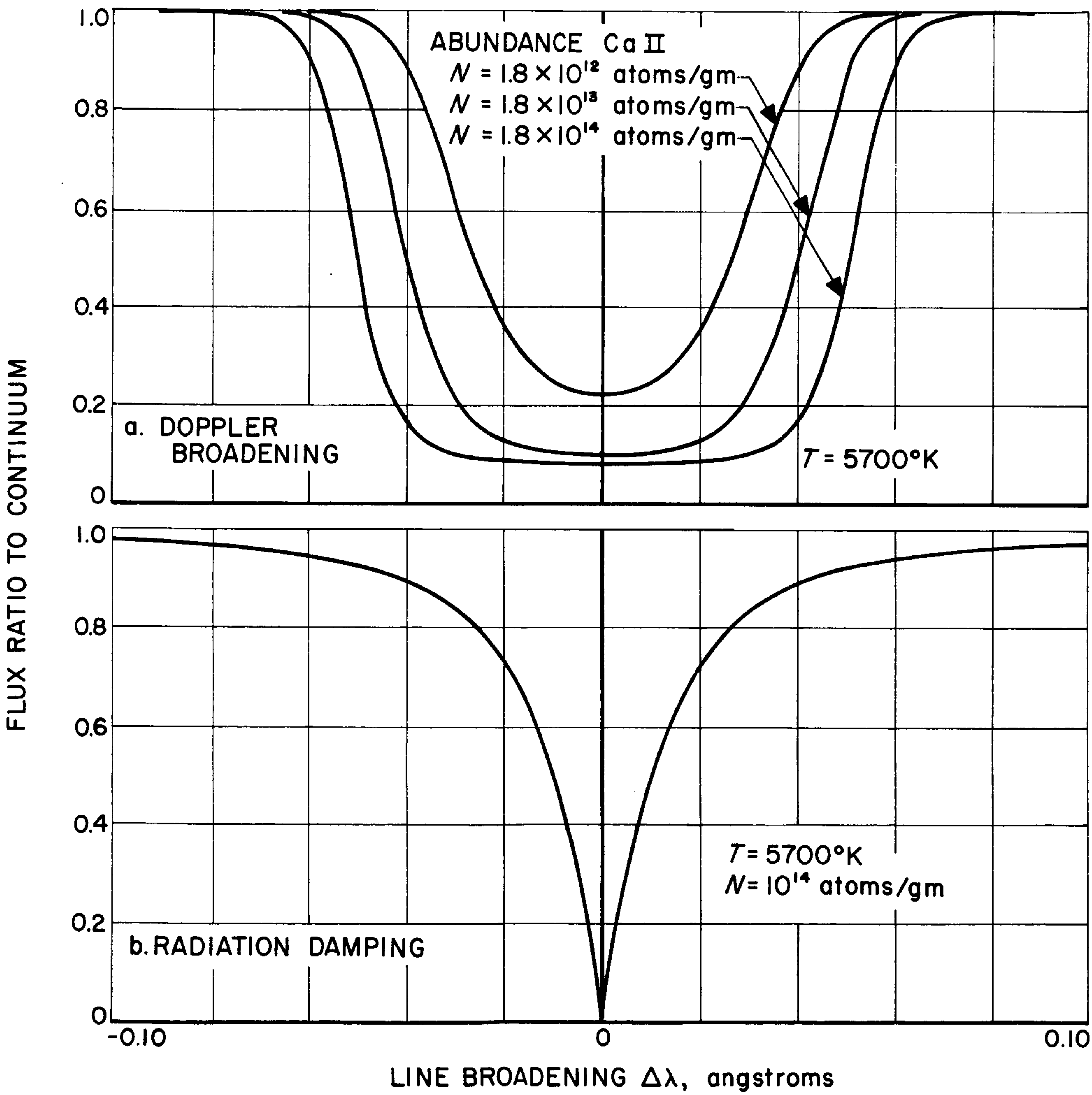

Fig. 1. Computed Spectral Line Profiles for Ca II, 3934 angstroms

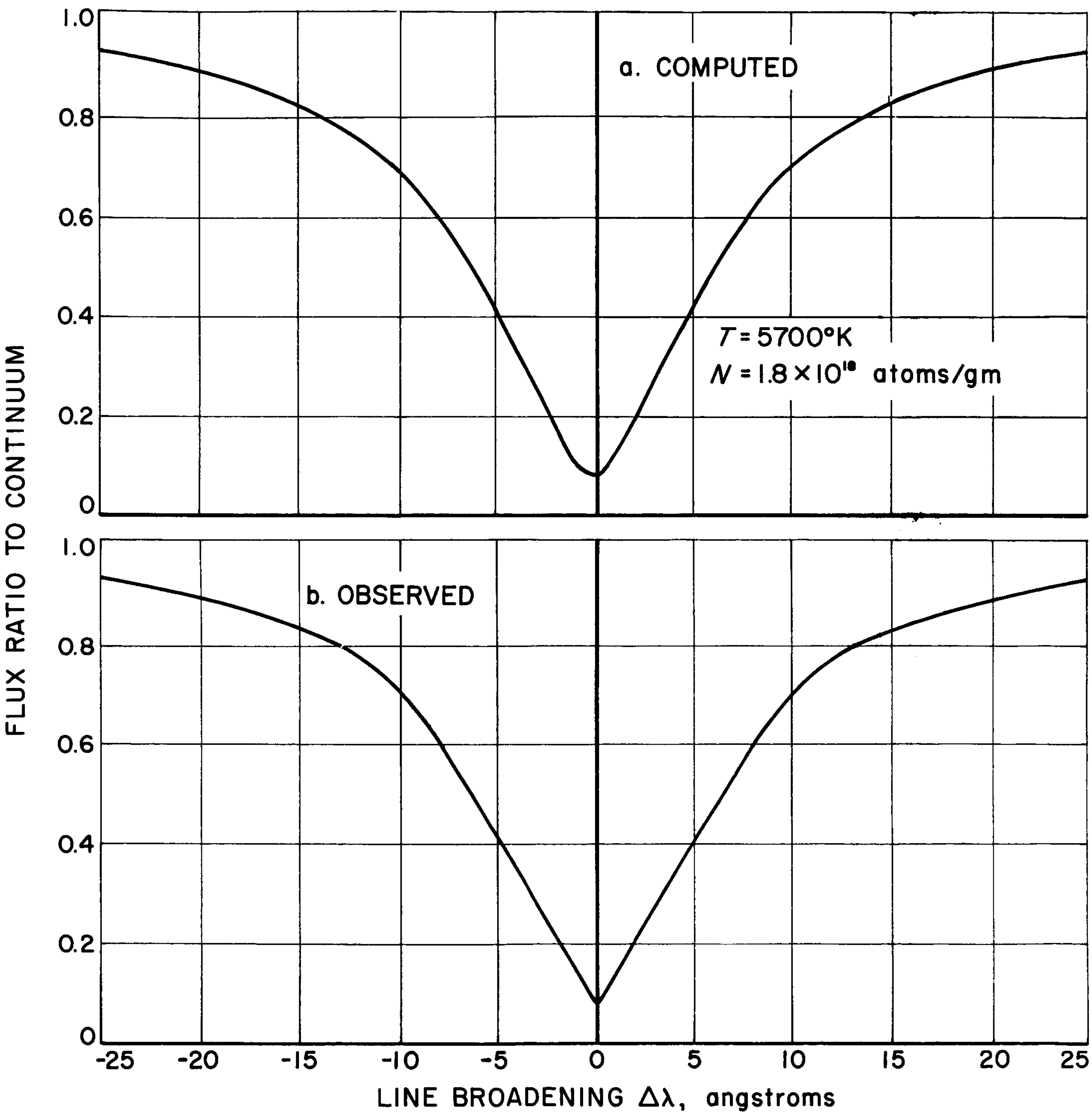

Fig. 2. Computed and observed solar line profiles for Ca II, 3934 angstroms including Doppler broadening, radiation damping, and collision damping

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